Int J Theor Phys (2008) 47: 2011–2020
DOI 10.1007/s10773-007-9645-2

Quantum Anomaly and Hawking Radiation of Charged and Magnetized Reissner-Nordström de Sitter Black Hole

Yi-Wen Han · Xiao-Xiong Zeng · Shu-Zheng Yang

Received: 20 October 2007 / Accepted: 28 December 2007 / Published online: 15 January 2008

Abstract The recent work of Robinson and Wilczek that Hawking radiation can be determined by the compensating fluxes is extended to the charged and magnetized Reissner-Nordström de Sitter black hole. We reconstruct the electromagnetic field tensor and the Lagrangian of the field corresponding to the source with electric and magnetic charges to redefine an equivalent charge and gauge potential. We construct the effect field theory between the event horizon and cosmological horizon to respectively determine the compensating fluxes from them, which are shown to exactly equal to those of Hawking radiation, by the covariant anomaly cancellation conditions.

Keywords Hawking radiation · Covariant anomaly · Black hole · Magnetic charge

1 Introduction

In 2005, Robinson and Wilczek proposed a new derivation of Hawking radiation [1], which ties its existence to the cancellation of gravitational anomaly at the horizon. As explained there, this derivation has important advantage; it localizes the source of the anomaly at the horizon, where the geometry is nonsingular yet equations simplify. And it can be applied to arbitrary dimensional space time, which means any higher dimensional space time can be reduced to the effective 2-dimensional theory by the universal dimensional reduction technique. Till now, via this method, a number of static and stationary black holes have been studied (see [2–10] and references therein). All of them disclosed the same result that

Y.-W. Han (✉)
Computer Science of College, Chongqing Technology and Business University, Chongqing 400067, People's Republic of China
e-mail: hanyw1965@163.com

X.-X. Zeng · S.-Z. Yang
Institute of Theoretical Physics, China West Normal University, Nanchong 637002, Sichuan, People's Republic of China

X.-X. Zeng
e-mail: xxzengphysics@163.com

the compensating fluxes of gauge and energy momentum tensor have an equivalent form to those of Hawking radiation. In this paper, we attempt to extend this method to discuss Hawking radiation from the charged and magnetized Reissner-Nordström de Sitter black hole. Because there not only exist gauge flux of electric charge but also exist gauge flux of magnetic charge, its quantum anomaly hasn't been treated of till now.

As a matter of fact, for the space-time with electric and magnetic charges, the outside of the hole is an electromagnetic vacuum, and the hole can be taken as a conducting sphere [11]. As a result, we can reconstruct the electromagnetic tensor and Lagrangian of the field to redefine an equivalent charge corresponding to the electric and magnetic charges. For the sake of simplicity here, we consider the rate of electric and magnetic charges of the outgoing modes is constant and equals that of the black hole. We find that the Lagrangian of the electromagnetic field can be expressed by a set of generalized coordinates $\tilde{A}_\mu = (\tilde{A}_t, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$, which are cyclic coordinates. Meanwhile, the nonvanishing gauge potential is given. The result shows both the equivalent charge and gauge potential are similar to that of the field only with electric charge. In this way, the anomaly cancellation method is applicable. It should be pointed out that the flux of Hawking radiation here is determined only by the covariant gauge and gravitational anomalies rather than the consistent one. The reason for this stems from the functional form of covariant anomaly is unique, being governed solely by the gauge or diffeomorphism transformation properties, but not so for the consistent one. This covariant anomaly cancellation method is believed conceptually to be clean and economical.

The remainder of this paper is outlined as follows. In Sect. 2, we reconstruct the electromagnetic field tensor and Lagrangian to redefine the equivalent charge and gauge potential corresponding to the source with electric and magnetic charge. Then in Sect. 3 and Sect. 4, we respectively determine Hawking radiation from the event horizon and cosmological horizon of charged and magnetized Reissner-Nordström de Sitter black hole by the covariant gauge and gravitational anomalies. Finally, ends up with our concluding remarks in Sect. 5.

2 Maxwell Equation and Lagrangian Function of Charged and Magnetized Reissner-Nordström de Sitter Black Hole

The charged and magnetized Reissner-Nordström de Sitter black hole is the exact solution of the Einstein field equation, which can be derived from the Lagrangian action [12]

$$A = \int \sqrt{-g}(16\pi G_0 L_m + R - H_{\mu\nu}H^{\mu\nu} + 2\lambda + F_{\mu\nu}F^{\mu\nu} + \omega K_\mu K^\mu R + \eta K^\mu K^\nu R_{\mu\nu}) d^4x, \quad (1)$$

with the conditions $\omega = \eta = 0$, $c = G_0 = 1$ where $H_{\mu\nu} = K_{\mu;\nu} - K_{\nu;\mu}$ in analogy with electrodynamics, ω and η are dimensionless parameter, $F_{\mu\nu}$ is the Maxwell tensor, λ and G_0 are the cosmological and gravitational constants. Its explicit expression takes the form as

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2+q^2}{r^2} - \frac{\lambda r^2}{3}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2+q^2}{r^2} - \frac{\lambda r^2}{3}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (2)$$

where Q and q are electric charge and magnetic charge of the celestial body. Because this black hole is charged and magnetized, it's inconvenient for us to discuss the gauge anomaly with respect to electric charge and magnetic charge directly. To adopt the covariant cancellation conditions, one should find an equivalent charge of the electric and magnetic charges firstly.

For a source with electric and magnetic charges, the electromagnetic tensor is defined as [13, 14]

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^+_{\mu\nu}, \tag{3}$$

where $G^+_{\mu\nu}$ is the Dirac string term. Correspondingly, the Maxwell equations can be rewritten as

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_e u^\mu, \tag{4a}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_q u^\mu, \tag{4b}$$

where $F^{+\mu\nu}$ is the dual tensor of$F^{\mu\nu}$, ρ_e and ρ_q are the densities of electric and magnetic charges, respectively, while u^μ is the 4-velocity. In order to rewrite the Maxwell equations (4a) and (4b) to a simple form, we define a new real anti-symmetric tensor

$$\tilde{F}^{\mu\nu} = F^{\mu\nu}\cos\alpha + F^{+\mu\nu}\sin\alpha, \tag{5}$$

where α denotes a real constant angle. Incorporating with (4a) and (4b), we get

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi(\rho_e\cos\alpha + \rho_q\sin\alpha)u^\mu, \tag{6a}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 4\pi(-\rho_e\sin\alpha + \rho_q\cos\alpha)u^\mu. \tag{6b}$$

Letting

$$\rho_e\cos\alpha + \rho_q\sin\alpha = \rho_h, \tag{7a}$$

$$-\rho_e\sin\alpha + \rho_q\cos\alpha = 0. \tag{7b}$$

Besides yields $\rho_e/\rho_q = \cot\beta$, the Maxwell equations can be modified as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi\rho_h, \tag{8a}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 0. \tag{8b}$$

That is

$$\frac{\partial}{\partial x^\nu}(\sqrt{-g}\tilde{F}^{\mu\nu}) = 4\pi\sqrt{-g}I^\mu, \tag{9}$$

where $I^\mu = \rho_h u^\mu$. Obviously, (9) is similar to the Maxwell equation corresponding to the source only with electric charge. Therefore, if we consider the black hole as a conducting sphere while the electric and magnetic charges are concentrated on the black hole with the density rate as $\rho_e/\rho_q = \cot\alpha$, we have

$$Q_h^2 = Q^2 + q^2, \tag{10}$$

where Q and q are the electric and magnetic charges of the hole, respectively, while Q_h is the equivalent charge corresponding to the density ρ_h. Similarly, we can construct the Lagrangian of the electromagnetic field as

$$L_h = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{11}$$

In (11) the corresponding generalized coordinates are

$$\tilde{A}_\mu = (\tilde{A}_t, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3), \tag{12}$$

which satisfy

$$\tilde{F}_{\mu\nu} = \nabla_\nu \tilde{A}_\mu - \nabla_\mu \tilde{A}_\nu. \tag{13}$$

Considering (10), the line element (2) now can be rewritten as

$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega^2, \tag{14}$$

where $f(r)$ is

$$f(r) = 1 - \frac{2M}{r} + \frac{Q_h^2}{r^2} - \frac{\lambda r^2}{3}. \tag{15}$$

According to the null hyper surface equation, this can yield the outer horizon r_e, inner horizon r_i and the cosmological horizon r_c. And the corresponding non-vanishing component of the electromagnetic vector potential can be expressed as

$$\tilde{A}_t = -\frac{Q_h}{r}. \tag{16}$$

While the surface gravity at the EH and CH can be respectively represented as [15]

$$\kappa_i = \frac{1}{2}\partial_r f|_{r=r_i} = (\lambda/6) r_i^{-2}(r_i - r_-)(r_e - r_i)(r_c - r_i), \tag{17}$$

$$\kappa_e = \frac{1}{2}\partial_r f|_{r=r_e} = (\lambda/6) r_e^{-2}(r_e - r_-)(r_e - r_i)(r_c - r_e), \tag{18}$$

$$\kappa = \frac{1}{2}\partial_r f|_{r=r_c} (\lambda/6) r_c^{-2}(r_c - r_-)(r_c - r_i)(r_c - r_e). \tag{19}$$

Due to observers are located between the outer-horizon and the cosmological horizon, what they observed only originate from them but not from the inner horizon, thus, we next only take into account the contributions of the event horizon and cosmological horizon to discuss the flux of Hawking radiation.

3 Quantum Anomaly at Black Hole Horizon

Now, we concentrate on studying the quantum anomaly at horizon. As mentioned above, before the effective field theory is constructed, one should reduce the arbitrary dimensional space time to an effective two dimensional theory via considering the action of scalar field in the black hole background. In the case of the charged and magnetized Reissner-Nordström de Sitter black hole, after performing the scalar field decomposition and tortoise coordinate transformation, we find physics that near the event horizon can be described by an infinite collection of massless 2-dimensional scalar field effectively on the metric

$$ds^2 = -f dt^2 + f^{-1} dr^2. \tag{20}$$

In the effective two-dimensional reduction, each partial wave mode has both gauge and general coordinate symmetries, when we integrate out the classically insignificant ingoing modes to define the effective field outside the event horizon; it becomes chiral and suffers from gauge and gravitational anomalies with respect to the gauge and general coordinate symmetries respectively. For the gauge flux at the event horizon, the consistent form of the two-dimensional Abelian anomaly [16] is given by

$$\Delta_\mu J^\mu = \pm \frac{e^2}{4\pi\sqrt{-g}} \varepsilon^{\mu\nu} \partial_\mu A_\nu, \tag{21}$$

where $+$ $(-)$ correspond to the left (right) handed field, $\varepsilon^{\mu\nu}$ is the anti-symmetric tensor $(\varepsilon^{10} = \varepsilon^{01} = -1)$. The covariant current and its relevant covariant anomalous equation [1] are

$$\tilde{J}^\mu = J^\mu \mp \frac{e^2}{4\pi\sqrt{-g}} \varepsilon^{\mu\nu} \partial_\mu A_\nu, \qquad \Delta_\mu \tilde{J}^\mu = \pm \frac{e^2}{4\pi\sqrt{-g}} \varepsilon_{\mu\nu} F^{\mu\nu}. \tag{22}$$

As stated in [17], the consistent anomalous equation of the consistent current is only the minimal form, which can't be expressed as the gauge variation of a local polynomial as in [1], but the functional forms of the covariant anomalies are unique. Thus we next don't adopt the initial approach of Robinson and Wilczek but employ the simplified method of canceling the covariant gauge and gravitational anomalies. For convenience, we divide the region Outside the event horizon into $[r_e, r_e+\varepsilon]$ and $[r_e+\varepsilon, +\infty]$ besides write the covariant flux as $\tilde{J}^\mu_{(2)} = \tilde{J}^u_{(0)}\Theta_+(r) + \tilde{J}^\mu_{(H)} H(r)$ by the scalar step function $\Theta_+ = \Theta(r - r_e - \varepsilon)$ and scalar top hat function $H = 1 - \Theta_+$. When the quantum effect of the classically irrelevant ingoing modes are omitted, there isn't anomaly in the region $[r_e+\varepsilon, +\infty]$ while exhibits one in the near-horizon region $[r_e, r_e + \varepsilon]$. The relevant covariant gauge fluxes thus respectively satisfy

$$\Delta_r \tilde{J}^r_{(H)} = \frac{e^2}{4\pi} F^{rt} = \frac{e^2}{2\pi} \partial_r A_t, \quad \partial_r \tilde{J}^r_{(0)} = 0. \tag{23}$$

Integrating (22) yields

$$\tilde{J}^r_{(0)} = a_0, \qquad \tilde{J}^r_{(H)} = a_h + e^2[A_t(r) - A_t(r_e)]/2\pi \tag{24}$$

in which a_0 and a_h are the integration constants, which stand for the values of the covariant fluxes of electric charge at infinity and the event horizon. Outside the event horizon, the Ward identity is

$$\Delta_\mu \tilde{J}^\mu = \partial_r \tilde{J}^r = \partial_r \left(\frac{e^2}{2\pi} A_t H \right) + (\tilde{J}^r_{(0)} - \tilde{J}^r_{(H)} + e^2 A_t(r)/2\pi)\delta(r - r_e - \varepsilon) \tag{25}$$

in which, the first term in the right will be counteracted by the quantum effect of the classically insignificant ingoing modes that we have ignored. This is the Wess-Zumino term that induced by the ingoing modes which contribute to the total gauge flux $-e^2 A_t H/2\pi$ at the event horizon. Hence only the coefficients of the delta function vanished can hold the gauge covariant at the quantum, which means

$$a_0 = a_h - e^2 A_t(r_e)/2\pi. \tag{26}$$

Imposing the vanishing condition of the covariant flux at the event horizon, from (24), we get $a_h = 0$. The electric charge current, which is required to cancel the gauge anomaly to restore the underlying gauge symmetry at the event horizon, thus is

$$a_0 = e^2 Q_h/2\pi r_e. \tag{27}$$

This current will be showed to equal to that of Hawking radiation with appropriate chemical potential precisely.

Next, we turn to studying the flux of energy-momentum tensor. When the classically irrelevant ingoing modes are excluded to construct an effective field outside the event horizon, due to the pileup of outgoing high frequency modes, the energy-momentum tensor is divergent which results in gravitational anomaly exhibits that takes the form of the breakdown of the conservation of energy-momentum tensor. For right handed field, the minimal form of the $(1+1)$-dimensional consistent anomaly reads [18–21]

$$\nabla_\mu T^\mu_\nu \equiv A_\nu \equiv \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_\nu, \tag{28}$$

where

$$N^\mu_\nu = \frac{1}{96\pi}\varepsilon^{\beta\mu}\partial_\alpha\Gamma^\alpha_{\nu\beta}. \tag{29}$$

And the covariant anomaly, on the other hand, also takes the form as [18–21]

$$\nabla_\mu \tilde{T}^\mu_{(H)\nu} \equiv \tilde{A}_\nu \equiv \frac{1}{\sqrt{-g}}\partial_\mu \tilde{N}^\mu_\nu. \tag{30}$$

For the two-dimensional space-time, the covariant anomaly is purely time-like ($\tilde{A}_r = 0$) while

$$\tilde{A}_t \equiv \partial_r \tilde{N}^r_t, \quad \tilde{N}^r_t = [2f(r)f''(r) - (f'(r))^2]/192\pi. \tag{31}$$

Since the existence of electric field, the covariant energy-momentum tensor isn't conserved even classically but keeps to the Lorenz force law $\nabla_\mu \tilde{T}^\mu_{\nu(0)} = F_{\mu\nu}\tilde{J}^\mu_{(0)}$ in the region $[r_e+\varepsilon,\infty]$. While in the region $[r_e, r_e+\varepsilon]$, adding the covariant gravitational anomaly that stems from the divergent energy-momentum tensor, the Ward identity changes as

$$\nabla_\mu \tilde{T}^\mu_{\nu(H)} = F_{\mu\nu}\tilde{J}^\mu_{(H)} + \tilde{A}_\nu, \tag{32}$$

Solving the Ward identity for the $\nu = t$ component in each region by (24), we find

$$\tilde{T}^r_{t(0)} = e_0 + a_0 A_t. \tag{33}$$

$$\tilde{T}^r_{t(H)} = e_H + \int_{r_e}^r dr\,\partial_r\left(a_0 A_t + \frac{e^2}{4\pi}A_t^2 + \tilde{N}^r_t\right), \tag{34}$$

where e_0 and e_H are integration constants. Then writing the covariant flux of energy-momentum tensor as $T^\mu_\nu = T^u_{\nu(0)}\Theta_+(r) + T^\mu_{\nu(H)}H(r)$ by the scalar step function and scalar top hat function, the Ward identity outside the event horizon can be expressed as

$$\begin{aligned}\nabla_\mu \tilde{T}^\mu_t = \partial_r \tilde{T}^r_t &= a_0\partial_r A_t(r) + [\partial_r(e^2A_t^2(r)/4\pi + \tilde{N}^r_t)H] \\ &\quad + (\tilde{T}^r_{t(0)} - \tilde{T}^\mu_{t(H)} + e^2A_t^2/4\pi + \tilde{N}^r_t)\delta(r - r_e - \varepsilon),\end{aligned} \tag{35}$$

where, the first term is the classical effect arises from the Lorenz force, the second term has to be canceled by the quantum effect of the classically irrelevant ingoing modes that contribute to the total energy-momentum tensor is $-(e^2A_t^2(r)/4\pi + \tilde{N}_t^r)$. That is, only the coefficients of the delta function satisfy

$$e_0 = e_H + \frac{e^2}{4\pi}A_t^2(r_e) - \tilde{N}_t^r(r_e), \tag{36}$$

can restore the diffeomorphism invariance at the event horizon. To get the explicit value of e_0, we impose the vanishing condition that requires the flux of covariant energy-momentum tensor to vanish at the event horizon. After this, the total energy-momentum tensor, which can cancel the gravitational anomaly at the event horizon to save the general coordinate covariance at the quantum level, can be written as

$$e_0 = \frac{e^2}{4\pi}A_t^2(r_e) - \tilde{N}_t^r(r_e) = \frac{e^2Q_h^2}{4\pi r_e^2} + \frac{\pi T_e^2}{12}, \tag{37}$$

where

$$T_e = \frac{\kappa_e}{2\pi} = \frac{1}{2\pi}(\lambda/6)r_e^{-2}(r_e - r_-)(r_e - r_i)(r_c - r_e), \tag{38}$$

is the Hawking temperature of the Reissner-Nordström de Sitter black hole.

Now, we focus on exploring the relation between the compensating fluxes and those of Hawking radiation. At the event horizon, the Planckian distribution with Hawking temperature as in (37) takes the form as

$$N_\pm(\omega) = \frac{1}{\exp[(\omega \pm eA_t(r_e))/T_e] \pm 1}. \tag{39}$$

For fermions, we define F_a and F_e as the Hawking radiation fluxes of electric charge and energy-momentum tensor and find

$$F_a = \int_0^\infty \frac{e}{2\pi}[N_e(\omega) - N_{-e}(\omega)]d\omega = \frac{e^2Q_h}{2\pi r_e}. \tag{40}$$

$$F_e = \int_0^\infty \frac{\omega}{2\pi}[N_e(\omega) + N_{-e}(\omega)]d\omega = \frac{e^2Q_h^2}{4\pi r_e^2} + \frac{\pi T_e^2}{12}. \tag{41}$$

Comparing (26) and (37) with (40) and (41), we find the fluxes of gauge and energy-momentum tensor, which required cancelling the gauge anomaly gravitational anomalies at the event horizon to restore the gauge and general covariance at the quantum level, precisely equal to that of a $(1 + 1)$-dimensional blackbody at the Hawking temperature with appropriate chemical potential.

4 Quantum Anomaly at the de Sitter Cosmological Horizon

In previous section, we omitted the anomaly effect at cosmological horizon to discuss Hawking radiation from the event horizon and found the compensating fluxes can hold the gauge and general covariance at the quantum level by canceling the gauge and gravitational anomalies at the event horizon. Next, we will incorporate with the quantum effect of the ingoing modes at the event horizon to discuss the Hawking radiation from cosmological horizon via the anomaly cancellation conditions. Note that in de-Sitter spaces, future infinity is space-like, which means any physics are only taken place inside the cosmological horizon. Thus, when the quantum effects at the event horizon are taken into account, we integrate out the classically irrelevant outgoing modes to construct the effective field inside the cosmological horizon. Quantum mechanics whereas tells us the contribution of outgoing modes at the cosmological horizon cannot be neglected. Therefore, under the gauge and general coordinate transformation, the effective field becomes chiral and exhibits gauge and gravitational anomaly. For gauge anomaly, near the cosmological horizon $r_c - \varepsilon \le r \le r_c$, due to non-symmetry of the ingoing modes and outgoing modes, there exhibits an anomaly which follows (for left handed field)

$$\Delta_\mu \tilde{J}^\mu = -\Delta_r \tilde{J}^r_{(C)} = -e^2 F^{rt}/4\pi = -(e^2/2\pi)\partial_r A_t. \tag{42}$$

While in the region $r \le r_c - \varepsilon$, there isn't anomaly and the covariant current satisfies

$$\partial_r \tilde{J}^\mu_{(0)} = \partial_r \tilde{J}^r_{(0)} = 0. \tag{43}$$

Solving (42) and (43) yields

$$\tilde{J}^r_{(0)} = c_0, \qquad \tilde{J}^r_{(C)} = c_c - e^2[A_t(r) - A_t(r_c)]/2\pi, \tag{44}$$

where c_0 and c_c are the integration constants, which respectively stand for the values of covariant fluxes of electric charge at infinity and the cosmological horizon. The Ward identity inside the cosmological horizon is

$$\Delta_\mu \tilde{J}^\mu = \partial_r \tilde{J}^r = -\partial_r(e^2 A_t H/2\pi) + \left(\tilde{J}^r_{(C)} - \tilde{J}^r_{(0)} + e^2 A_t(r)/2\pi\right)\delta(r - r_C + \varepsilon) \tag{45}$$

in which, we have written the covariant flux inside the cosmological horizon as $\tilde{J}^\mu = \tilde{J}^u_{(0)}\Theta_-(r) + \tilde{J}^\mu_{(C)}C(r)$, where $\Theta_- = \Theta(r_c - r - \varepsilon)$ and $C = 1 - \Theta_-$ are scalar step function and scalar top hat function respectively. Similarly when the quantum effect of the outgoing modes that we have omitted is taken into account, the properties of the delta function will impose

$$c_0 = c_c + e^2 A_t(r_c)/2\pi. \tag{46}$$

After the vanishing condition is forced at the cosmological horizon, the total electric charge current read off

$$c_0 = -e^2 Q_h/2\pi r_c. \tag{47}$$

The negative sign denote the effective field must absorb the gauge flux, which will be showed to precisely equal to the flux of Hawking radiation from the cosmological horizon, to save the gauge invariance at the cosmological horizon of the Reissner-Nordström de Sitter black hole at the quantum level.

Besides gauge symmetry, the effective field for each partial wave inside the cosmological horizon also exhibits general coordinate symmetry. When the classically irrelevant outgoing modes are excluded at the cosmological horizon, the general coordinate symmetry is breakdown which leads to the covariant energy-momentum tensor fail to conserve in $r_c - \varepsilon \le r \le r_c$. That is, the covariant fluxes in the cosmological horizon should respectively satisfy the following conservation and anomalous equation

$$\nabla_r \tilde{T}^r_{\nu(0)} = F_{r\nu}\tilde{J}^r_{(0)}, \qquad \nabla_r \tilde{T}^r_{\nu(C)} = F_{r\nu}\tilde{J}^r_{(C)} - \tilde{A}_\nu. \tag{48}$$

Solving them by (44), we get

$$\tilde{T}^r_{t(0)} = d_0 + c_0 A_t, \qquad \tilde{T}^r_{t(H)} = d_C + \int_{r_c}^{r} dr\,\partial_r \left(c_0 A_t - \frac{e^2}{4\pi} A_t^2 - \tilde{N}_t^r \right), \tag{49}$$

where d_0 and d_C are the integration constants, which stand for the values of the covariant flux of energy-momentum tensor at infinity and the cosmological horizon. Employing the covariant conservation and anomalous equation, we find the Ward identity inside the cosmological horizon becomes as

$$\begin{aligned}\nabla_\mu \tilde{T}_t^\mu = \partial_r \tilde{T}_t^r = c_0 \partial_r A_t(r) - [\partial_r (e^2 A_t^2/4\pi A_t^2(r) + \tilde{N}_t^r) C] \\ + (\tilde{T}^r_{t(C)} + e^2 A_t^2/4\pi + \tilde{N}_t^r)\delta(r - r_c + \varepsilon).\end{aligned} \tag{50}$$

The first term is the classical effect of the background electric field for constant current flow, the second term has to be canceled by the quantum effects of the classically irrelevant outgoing modes that contribute to the total energy-momentum tensor is $(e^2 A_t^2(r)/4\pi + \tilde{N}_t^r)C$. Therefore, to hold the general coordinate invariance at the cosmological horizon, the coefficient of the delta function should be vanished, that is

$$d_0 = d_C - e^2 A_t^2(r_c)/4\pi - \tilde{N}_t^r(r_c). \tag{51}$$

To get the observable flux for us physically, we should first determine d_C. If the cosmological horizon is treated as the boundary of the de-Sitter space, then considering the covariant boundary condition at the cosmological horizon, we have $d_C = 0$. Thus the total energy-momentum tensor from the cosmological horizon can be observed by us is

$$d_0 = -e^2 A_t^2(r_c)/4\pi - \tilde{N}_t^r(r_c) = -e^2 Q_h^2/4\pi r_c^2 - \pi T_c^2/12, \tag{52}$$

where

$$T_c = \frac{\kappa_c}{2\pi} = \frac{1}{2\pi}(\lambda/6) r_c^{-2}(r_c - r_-)(r_c - r_i)(r_c - r_e) \tag{53}$$

is the Hawking temperature from the cosmological horizon of the Reissner-Nordström de Sitter black hole. At the cosmological horizon, the Planckian distributions of fermions with this temperature take the form as

$$N_{\pm e}(\omega) = -\frac{1}{\exp[(\omega \pm e A_t(r_c))/T_c] + 1}, \tag{54}$$

which satisfies

$$F_c = \int_0^\infty \frac{e}{2\pi}[N_e(\omega) - N_{-e}(\omega)] d\omega = -\frac{e^2 Q_h}{2\pi r_c}, \tag{55}$$

$$F_d = \int_0^\infty \frac{\omega}{2\pi}[N_e(\omega) + N_{-e}(\omega)] d\omega = -\frac{e^2 Q_h^2}{4\pi r_c^2} - \frac{\pi T_c^2}{12}. \tag{56}$$

From above, we find the flux of Hawking radiation from the cosmological horizon precisely equal to the compensating fluxes. The negative sign means to hold the underlying gauge and general coordinate invariance at the quantum level; the effective field should absorb the Hawking radiation flux from the cosmological horizon.

5 Conclusions

In the charged and magnetized Reissner-Nordström de Sitter space-time, there exist gauge currents with respect to electric charge and magnetic charge, it is inconvenient to save the underlying gauge symmetry by canceling the gauge anomaly directly. To overcome this difficulty, we reconstructed the electromagnetic tensor and Lagrangian to redefine an equivalent charge and gauge potential, which are similar to those of the field only with electric charge. In addition to describe the observable physics, the effective field is formulated between the event horizon and the cosmological horizon due to observers in the de Sitter space is located between them. Then, we employed the covariant anomaly cancellation method to determine the Hawking radiation. Our result is consistent and evolutive with the Robinson-Wilczek's initial viewpoint that Hawking radiation can be determined by the compensating fluxes at the event horizon and the cosmological horizon.

Acknowledgements This work was supported by the scientific and technological foundation of Chongqing Municipal Education Commission (Grant No.KJ0707011).

References

1. Robinson, S.P., Wilczek, F.: Phys. Rev. Lett. **95**, 011303 (2005)
2. Iso, S., Umetsu, H., Wilczek, F.: Phys. Rev. Lett. **96**, 151302 (2006)
3. Murata, K., Soda, J.: Phys. Rev. D **74**, 044018 (2006)
4. Setare, M.R.: Eur. Phys. J. C **49**, 865 (2006)
5. Iso, S., Morita, T., Umetsu, H.: J. High. Energy Phys. **04**, 068 (2007)
6. Shin, H., Kim, W.: J. High. Energy Phys. **06**, 012 (2007)
7. Xiao, K., Liu, W.B., Zhang, H.B.: Phys. Lett. B **647**, 482 (2007)
8. Wu, S.Q., Peng, J.J.: arXiv:hep-th/0706.0983 (2007)
9. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Rev. D **75**, 064029 (2007)
10. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **647**, 200 (2007)
11. Makela, J., Repo, P.: Phys. Rev. D **57**, 4899 (1998)
12. Gong, T.X., Wang, Y.J.: Chin. Phys. **14**, 0045 (2005)
13. Zhang, J.Y., Fan, J.H.: Phys. Lett. B **648**, 133 (2007)
14. Zhang, J.Y.: Mod. Phys. Lett. A **22**, 1821 (2007)
15. Liu, W.B., Zhao, Z.: J. Beijing Normal Univ. (Natural Science) **31**, 476 (2000)
16. Bertlmann, R.: Anomalies in Quantum Field Theory. Pergamon Press, Oxford (2000)
17. Banerjee, R., Kulkarni, S.: hep-th/0707.2449 (2007)
18. Bardeen, W.A., Zumino, B.: Nucl. Phys. B **244**, 421 (1984)
19. Witten, E.: Adv. Theoret. Mathemat. Phys. **2**, 253 (1998)
20. Bertlmann, R., Kohlprath, E.: Ann. Physics (New York) **288**, 137 (2001)
21. Hossain Ali, M.: arXiv:hep-th/0706.3890 (2007)